

## 1. INTRODUCTION

This document represents a report of technical progress during the contract entitled "Determination of Coronal Magnetic Fields from Vector Magnetograms," NASW-4728, between NASA and Science Applications International Corporation. Under this contract SAIC has conducted research into the determination of coronal magnetic fields from vector magnetograms, including the development and application of algorithms to determine force-free coronal fields above selected observations of active regions. The present contract began on June 30, 1992 and has a completion date of December 31, 1992. The basic contract has two additional one-year options to continue the contract through December 31, 1993 and December 31, 1994, respectively. It is our understanding that NASA intends to exercise Option 1 to continue the contract through December 31, 1993. Therefore, this report is to be considered a Progress Report for the present contract for work done during calendar year 1992. This contract is a continuation of work started in a previous contract, NASW-4571, which covered the period November 15, 1990 to December 14, 1991.

Since the present contract was funded at only a fraction of the requested amount, the research effort had to be scaled back from the anticipated level. Nevertheless, we have made substantial progress during the present contract period, including the selection and analysis of two additional active regions (October 24, 1991 and November 15, 1991, both sites of large flares). We have also made substantial progress in our code development effort, since we have removed the restriction of periodicity in our 3D code which is used to determine the coronal field. The new version of the code has variable mesh spacing and is thus able to provide a more realistic description of coronal fields.

We have continued to study NOAA active region AR5747 of 20 October, 1989, which is the original data set to which we have devoted much attention. The physical properties and flaring activity of this region have been studied extensively at the University of Hawaii.

All the vector magnetograms used so far have been obtained at Mees Solar Observatory of the University of Hawaii using the Haleakala Stokes Polarimeter. This project has benefited tremendously from cooperation with the staff of the Institute for Astronomy, University of Hawaii. Dr. Alexander McClymont has been our point-of-contact and principal collaborator during this project.

An oral paper by Dr. Zoran Mikić and Dr. Alexander McClymont entitled "Properties of Coronal Magnetic Fields Calculated from Vector Magnetograms" was presented at the meeting of the American Astronomical Society (Solar Physics Division) in Columbus, Ohio, June 8-11, 1991. A copy of the presentation appears in Appendix A.

Two papers resulting from research conducted under this contract (and the preceding contract) are presently in preparation. The titles of the two papers are: Mikić, Z., and Barnes, D. C., "Determination of Force-Free Coronal Fields from

Vector Magnetograms: I. Theoretical Basis," and Mikić, Z., and McClymont, A.N., "Determination of Force-Free Coronal Fields from Vector Magnetograms: II. Application to AR 5747 of 20 October, 1989," in preparation. We intend to publish these two papers in *The Astrophysical Journal*.

Section 2 of this report contains an brief account of progress during the research performed under this contract. Section 3 contains the proposed statement of work for the next year of the project.

## 2. TECHNICAL DETAILS

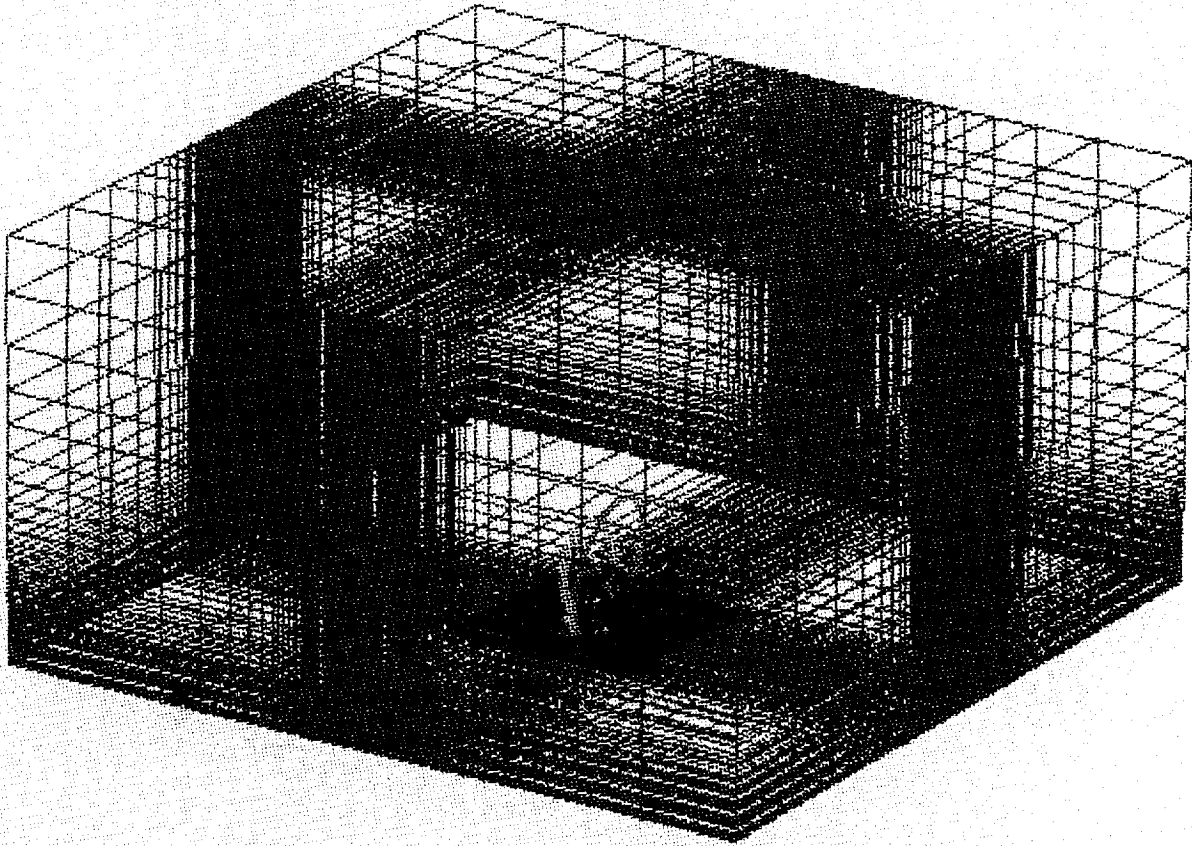
The technical details of the application of the evolutionary technique to determine coronal fields from vector magnetograms have been described previously (Z. Mikić, Final Report, NASW-4571, "Determination of the Coronal Magnetic Field from Vector Magnetograph Data," Dec. 14, 1991). The papers which describe the method and its application are in preparation, and have already been mentioned in Section 1. Appendix A contains a copy of the paper presented at the AAS/SPD meeting in Columbus, Ohio, in June, 1991. In this section we briefly state the latest developments.

### 2.1. New Non-Periodic Version of the 3D Code

Previously, our 3D code which was used to determine the coronal magnetic field assumed that the geometry was Cartesian, but that the two transverse dimensions ( $x$  and  $y$ ) were periodic. (Height in the corona is along the  $z$  dimension, with the plane  $z = 0$  representing the photosphere.) This assumption of periodicity required the active region to be isolated from the boundaries with a "guard region" to isolate it from its (unphysical) periodic neighbors, and required a uniform mesh in these transverse dimensions. Therefore, many mesh points were "wasted" in the guard region, where the fields are weak (and the flux is zero). Periodicity was assumed in order to simplify the numerical solution of the MHD equations, which can be done very efficiently along periodic coordinates using pseudo-spectral techniques.

During the past contract period we have implemented a new version of the code in which the transverse dimensions are *non-periodic*. This generalization has been possible through the use of a new field solver which is based on the preconditioned conjugate gradient method. It is now possible to invert 3D finite-differenced operators using this rapidly-converging iterative method. This allows the semi-implicit time-integration technique (which is required for the efficient solution of the MHD equations) to be carried out in 3D finite-difference geometry. In particular, the new code allows *non-uniform* meshes to be used. This allows us to increase the spatial resolution in the active-region where the field is strong, and to use larger mesh cells in the surrounding region where the field is weak. It is also possible to use large cells in the guard area to place the computational boundaries far away from the center of the active region, in order to minimize their effect on the accuracy of the computed coronal field. Figure 1 shows an example in which the use of non-uniform mesh cells allows the boundaries to be placed many scale lengths away, while still maintaining high resolution within the active region. The active-region area with strong fields is shown as the green area in the figure, along with the corresponding potential magnetic field lines.

This new capability will allow us to model the coronal magnetic field more realistically. We intend to re-compute the coronal field for the vector magnetogram of AR 5747 of October 20, 1989 with this version of the code.



**Figure 1. Field lines and flux distribution for a typical vector magnetogram. The colored area corresponds to the active region scale. Note how the non-uniform mesh can maximize resolution within the active region while placing the boundaries far away.**

## **2.2. Application to Two New Vector Magnetograms**

We have recently obtained (from U. Hawaii observers) two new vector magnetograms which have been processed to the extent to which we can attempt to compute the corresponding coronal fields. The two magnetograms were taken on October 24, 1991, and November 15, 1991. Both of these magnetograms were taken of active regions which experienced large flares. In particular, there exist Yohkoh soft X-ray observations of the corona above these active regions. These regions have also been studied at the University of Hawaii. Therefore, these two regions offer a possibility of comparing the computed fields with coronal observations. As stated in Section 3, we intend to make such comparisons during the next year of this project.

We have started processing the October 24, 1991 data. Figure 2 shows the vertical magnetic field in the photosphere, and the corresponding potential magnetic field. The vertical current density in the photosphere has also been computed. We are currently applying the evolutionary technique to this magnetogram.

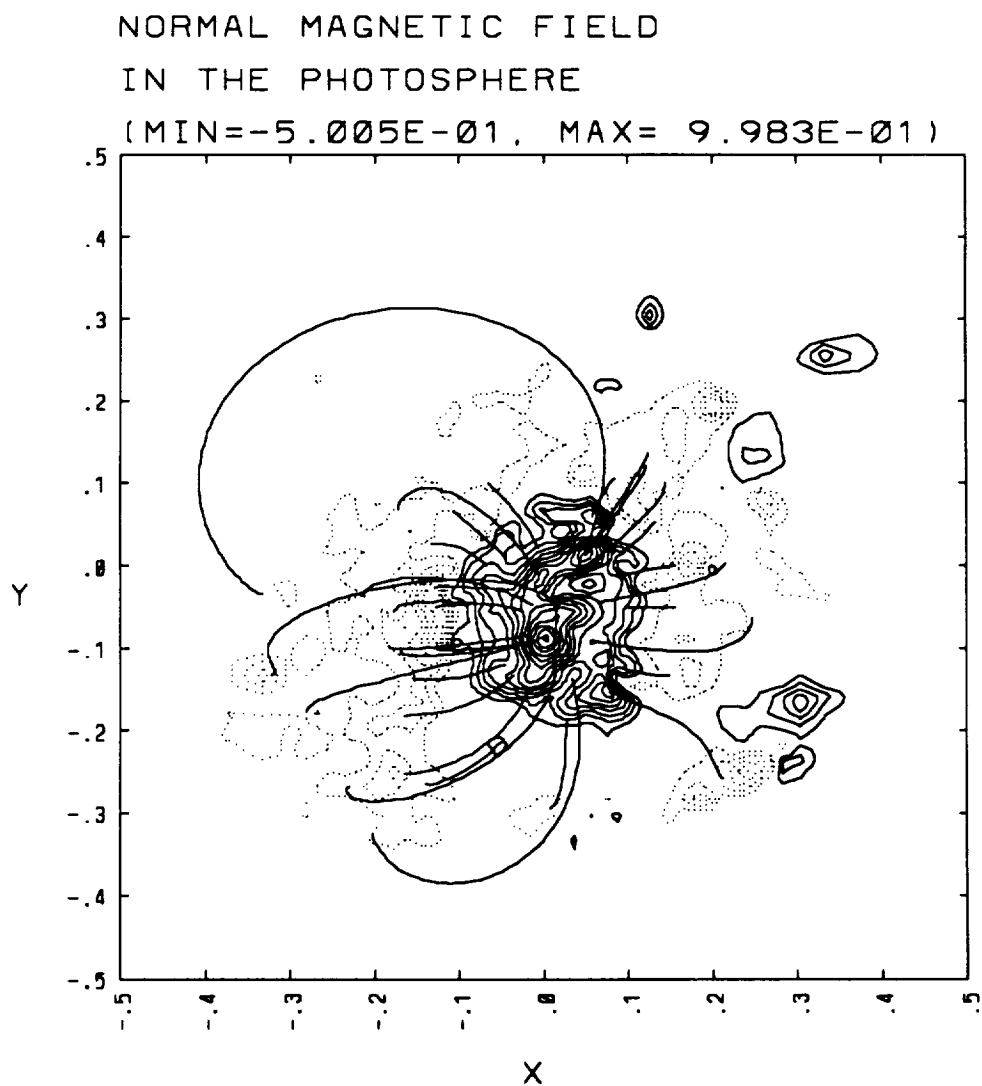


Figure 2. Vertical magnetic field contours (in the photosphere) and projections of the potential field lines corresponding to the vector magnetogram of October 24, 1991.

### **3. STATEMENT OF WORK (2ND YEAR)**

During the second year of this project we intend to continue development of the "evolutionary technique" for the determination of coronal magnetic fields. Next year's plan is centered around three principal goals:

#### **3.1. Assessment of the Accuracy of Computed Coronal Fields:**

We will concentrate on a careful examination of the accuracy and limitations of the evolutionary technique, especially to quantify the effects of measurement errors and the effect of limited spatial resolution on accuracy. The present magnetograms have shown that the measured magnetic fields are barely resolved (frequently the measured magnetic fields vary by tens of percent from one pixel to the next). The effect of errors will be studied by applying the technique to "synthesized magnetograms" which will be constructed by adding random errors to fields with known solutions.

#### **3.2. Application to Additional Active-Region Magnetograms:**

As described in Section 2, we are presently trying to determine the coronal magnetic fields above active regions observed on October 24, 1991 and November 15, 1991. These regions experienced large flares that were observed with Yohkoh, so they ought to be very interesting. We also intend to apply the method to several additional data sets, including those from the new Imaging Vector Magnetograph at the University of Hawaii, and the Applied Physics Laboratory Vector Magnetograph, which is currently operating at Sacramento Peak Observatory (SPO). The schedule for the Advanced Stokes Polarimeter, which is currently undergoing testing and calibration at SPO, indicates that it may be operational within the next year. This instrument has the capability to provide high-resolution vector magnetograms with very high accuracy. We plan to use data obtained from this instrument to determine coronal fields.

#### **3.3. Comparison of Computed Coronal Fields with Observations:**

We will make a concerted effort to identify vector magnetograms which correspond to simultaneous observations which can be used to identify coronal features, in order to validate the accuracy of computed coronal fields. In particular, soft X-ray observations provide images which seem to be very suggestive of magnetic features (loops, arcades, streamers). In this regard the recent high-resolution Yohkoh soft X-ray observations will be very useful. In particular, the two regions presently under study (Oct. 24, 1991 and Nov. 15, 1991) have corresponding Yohkoh soft X-ray data. This data will be used to identify

features which appear to trace magnetic field lines, which will be compared to the computed coronal fields which match vector magnetograms. In addition, we plan to collaborate with the staff of Lockheed, Palo Alto, (with Ted Tarbell as point-of-contact) to use coordinated measurements of H- $\alpha$  movies (which use downflows to identify coronal loops) and vector magnetograms made at La Palma Observatory, to determine if computed coronal fields match coronal features deduced from observations.

**APPENDIX A:**

PAPER PRESENTED AT THE AMERICAN ASTRONOMICAL SOCIETY  
(SOLAR PHYSICS DIVISION)  
MEETING, JUNE 8-11, 1992, COLUMBUS, OHIO

Properties of Coronal Magnetic Fields Calculated from Vector Magnetograms\*

Z. Mikić (SAIC, San Diego) and A. N. McClymont (IfA, U. Hawaii)

The “evolutionary technique” for finding coronal magnetic fields from vector magnetograms has been applied to a vector magnetogram of NOAA active region 5747 on October 20, 1989 taken with the Haleakala Stokes Polarimeter at Mees Solar Observatory, University of Hawaii. The evolutionary technique estimates the (non-constant- $\alpha$ ) force-free field in the corona by employing the time-dependent resistive MHD equations. The field is evolved from an initial potential field to a force-free field using an adaptive external circuit which drives the required electric current into the corona. When the field reaches a steady-state, it satisfies the force-free equations and matches the observed photospheric value of the magnetic flux and the inferred photospheric value of the normal current density. We have used this technique to estimate the coronal magnetic field for AR 5747. We will present the properties and expected accuracy of the estimated coronal field, an analysis of the vector magnetogram, as well as details on the performance of the evolutionary technique.

\*Research supported by NSF and NASA.

**PROPERTIES OF CORONAL  
MAGNETIC FIELDS  
CALCULATED FROM VECTOR  
MAGNETOGRAMS\***

**ZORAN MIKIĆ  
SAIC, SAN DIEGO**

**ALEXANDER N. MCCLYMONT  
IFA, U. HAWAII**

**\*Supported by NSF and NASA**

# INTRODUCTION

- We wish to find force-free fields ( $\mathbf{J} \times \mathbf{B} = 0$ ) in the corona
- Nonlinear force-free solution (i.e., non-constant  $\alpha$ )
- Boundary value problem formulation
- Use the “*evolutionary technique*”: the force-free field is the asymptotic state of a related time-dependent problem
- Applied to vector magnetogram data from the Stokes Polarimeter, IfA, U. Hawaii, for NOAA AR5747 of Oct. 20, 1989

## BOUNDARY CONDITIONS

- Schmidt (1968) conjectured that a solution exists when  $B_n$  and  $\alpha$  are specified on  $\partial\mathcal{D}$
- Given  $B_n$  on  $\partial\mathcal{D}$  and  $\alpha$  on  $\partial\mathcal{D}^+$ , the solution exists for small  $\alpha$  (Bineau 1972), and does not exist for large  $\alpha$  (Bineau 1972; Aly 1984, 1988, 1989)
- The FFF solution is stable with respect to changes in boundary conditions to the same extent that the potential field is (Molodensky 1974)
- We have confirmed that a solution can be found with these boundary conditions (3D sunspot field)
- Sakurai (1981) proposed an iterative solution technique which works for very small  $\alpha$  (based on Grad 1958)

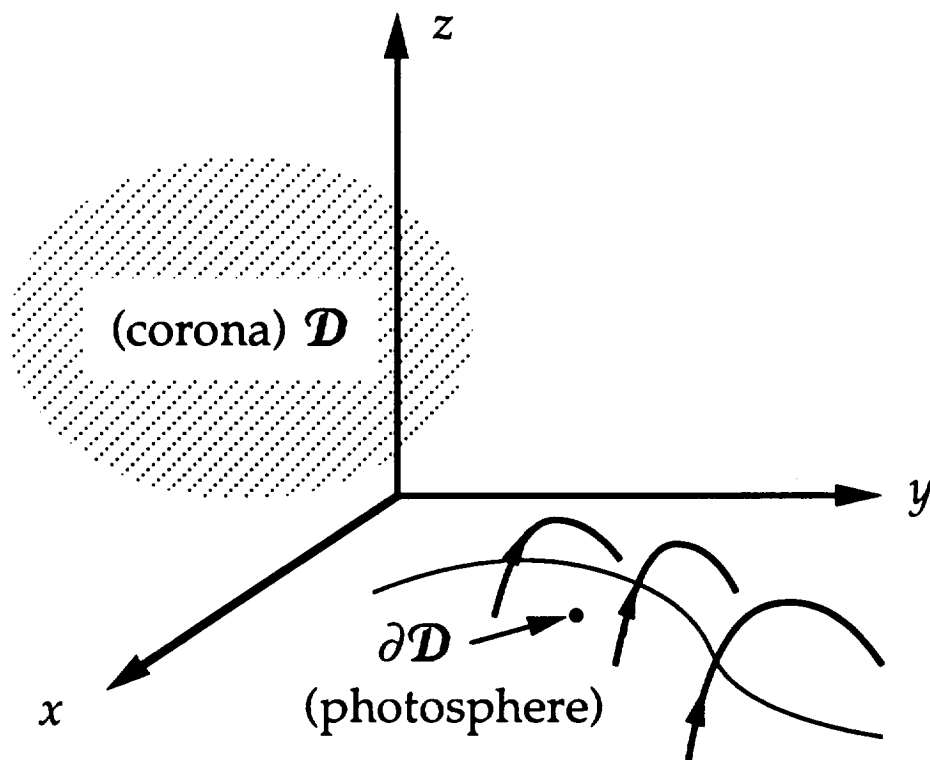
# FORCE-FREE CORONAL FIELDS

Let  $\mathcal{D} = \{z > 0\}$  (Planar approx.)

Find  $\mathbf{B}$  in  $\mathcal{D}$  satisfying

$$\nabla \times \mathbf{B} = \alpha \mathbf{B}$$

given  $\mathbf{B}$  on  $\partial\mathcal{D}$ .



## SOLUTION TECHNIQUE

- Apply boundary conditions on  $B_n$  and  $J_n$  (derived from VM data)
- For the given  $B_n$  on  $\partial\mathcal{D}$ , find the potential field  $\mathbf{B}_{\text{pot}}$  in  $\mathcal{D}$
- Apply a potential  $V(x,y,t)$  on  $\partial\mathcal{D}$  using an adaptive external circuit to force  $J_n = J_n^0$  (the measured normal current), while keeping  $B_n$  fixed
- Use the resistive MHD equations to evolve the plasma
- If a steady-state is reached, it will approach  $\mathbf{J} \times \mathbf{B} \approx 0$  in  $\mathcal{D}$ , and will have the required  $B_n$  and  $J_n$  on  $\partial\mathcal{D}$ .

# RESISTIVE MHD EQUATIONS

$$\nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{J}$$

$$\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}$$

$$\mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{B} = \eta \mathbf{J}$$

$$\rho \left( \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = \frac{1}{c} \mathbf{J} \times \mathbf{B} + \nu \rho \nabla^2 \mathbf{v}$$

## STEADY STATE SOLUTION

$$\nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{J}$$

$$\mathbf{J} \times \mathbf{B} = -c\rho \left( \nu \nabla^2 \mathbf{v} - \mathbf{v} \cdot \nabla \mathbf{v} \right)$$

$$\mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{B} = \eta \mathbf{J}$$

$$\nabla \times \mathbf{E} = 0$$

- Driven resistive MHD equilibrium with flow
- For small  $\eta$  and  $\nu$  (and hence  $\mathbf{v}$ ), the solution approaches a force-free state

## BOUNDARY ALGORITHM

- Impedance  $Z$ :

$$Z = \frac{\eta l}{d A}$$

- Choose a capacitance  $C$ :

$$C = v_o Z \Delta t$$

with  $v_o > 1$ ;  $v_o$  controls the time scale of the external circuit

- Evolve  $V$  according to:

$$\frac{\partial V}{\partial t} = \frac{d A}{C} (J_n^o - J_n)$$

- This gives  $E_t$  on the boundary:

$$E_t = -\nabla_t V$$

- Note that  $B_n$  remains fixed on the boundary (since  $\nabla_t \times E_t = 0$ )

# FEATURES

- Evolve  $\mathbf{B}$  toward a force-free state using resistive MHD
- Resistivity is essential to allow the topology to change during the evolution
- The dynamical equations allow ideal and resistive instabilities to occur (giving a minimum-energy final state subject to given boundary conditions?)
- Adaptive external circuit automatically forces the right current to flow in the corona

## APPLICATION TO AR5747 (OCT. 20, 1989)

- Use Lites-Skumanich code to fit Stokes profiles, and deduce **B**
- Resolve  $180^\circ$  ambiguity in transverse **B** (Tom Metcalf)
- Rotate and translate data to best fit a rectangular region [for AR5747, rotate by  $-27^\circ$  about (25,25) pix.]
- Interpolation/extrapolation of data: least squares fit to Fourier series, with derivative constraints
- Run resistive MHD code [used a  $63 \times 64 \times 64$  mesh and a  $63 \times 128 \times 128$  mesh on a Cray-2]
- Converges to a steady state in  $O(100\tau_A)$

## PROPERTIES OF THE VM

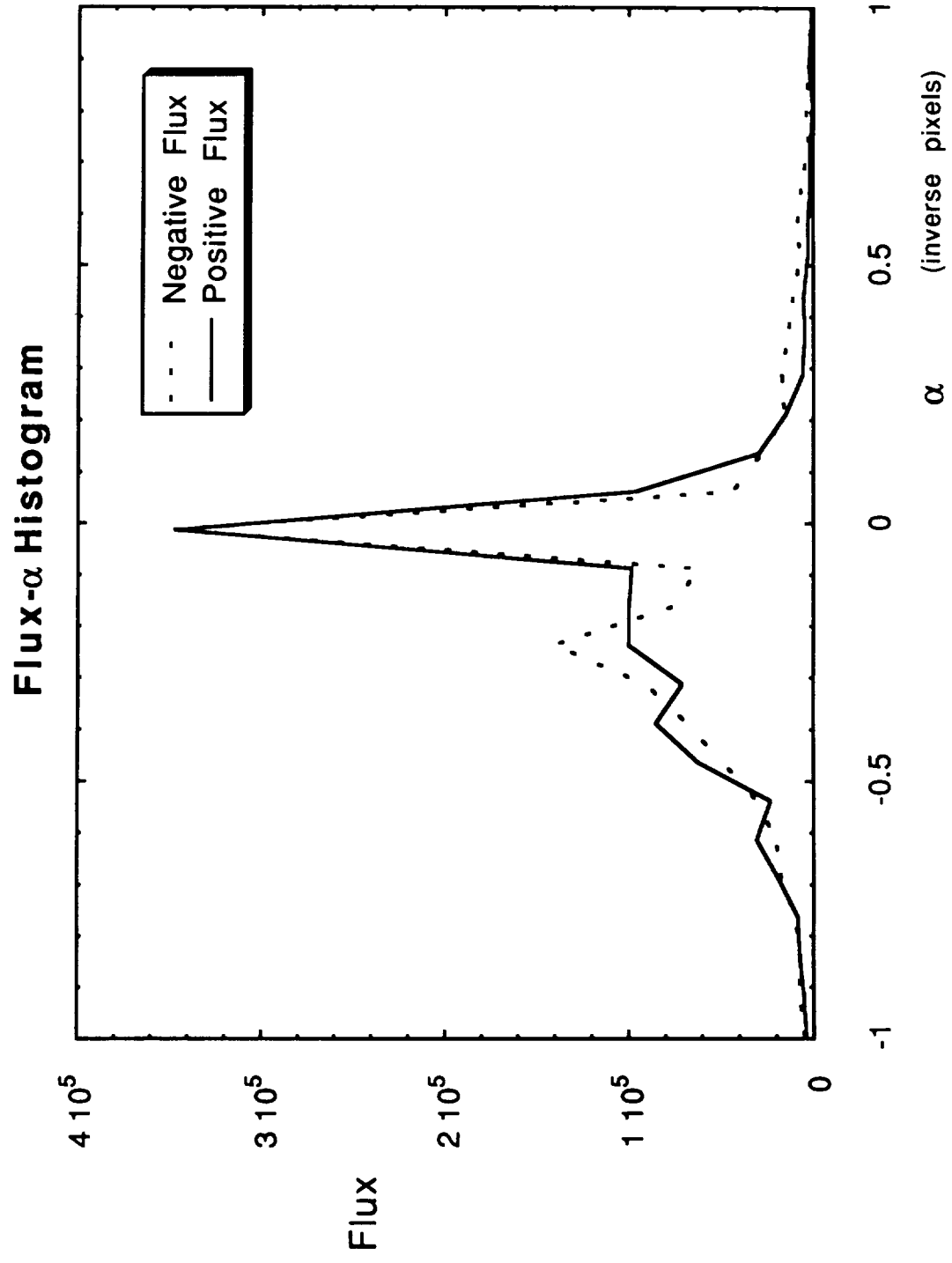
- Expected measurement errors in  $B$  are ( $B_{max} \sim 3000\text{G}$ ):
  - error in  $B_{los} \sim 20 - 50 \text{ G}$
  - error in  $B_{trans} \sim 200 - 300 \text{ G}$
- Due to projection effects (off disk center), the large errors in  $B_{trans}$  corrupt both the vertical and the horizontal fields
- The errors introduced due to finite resolution ( $30 \times 30$  pixels @  $5.7''$  per pixel) are unknown
- For a force-free field, at the neutral line ( $B_n = 0$ )  $J_n$  ought to be zero
- The VM data indicates that  $J_n$  is not negligible near the neutral line (up to 20% – 30% of the maximum)

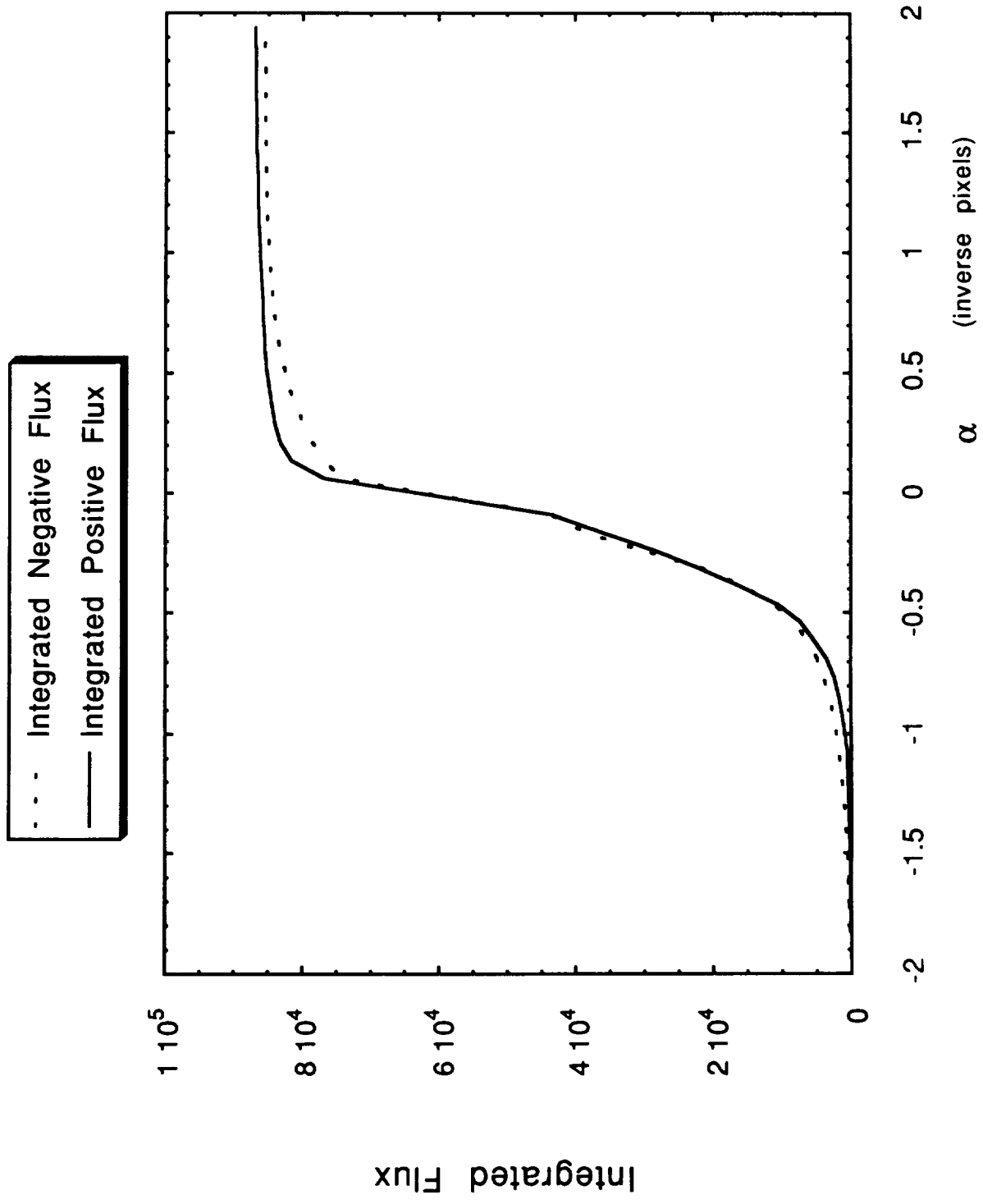
- The measurements are barely resolved (if that): e.g.,  $J_n$  changes by up to 50% between one pixel and the next
- Total flux balance is good ( $\sim 2\%$  over the magnetogram area)
- Flux balance as a function of  $\alpha$  is not as good. We compute:

$$\Phi_+(\alpha) = \int_{\alpha}^{\alpha+\Delta\alpha} B_n dS \quad (\text{for } B_n > 0)$$

$$\Phi_-(\alpha) = \int_{\alpha}^{\alpha+\Delta\alpha} B_n dS \quad (\text{for } B_n < 0)$$

(see figure)

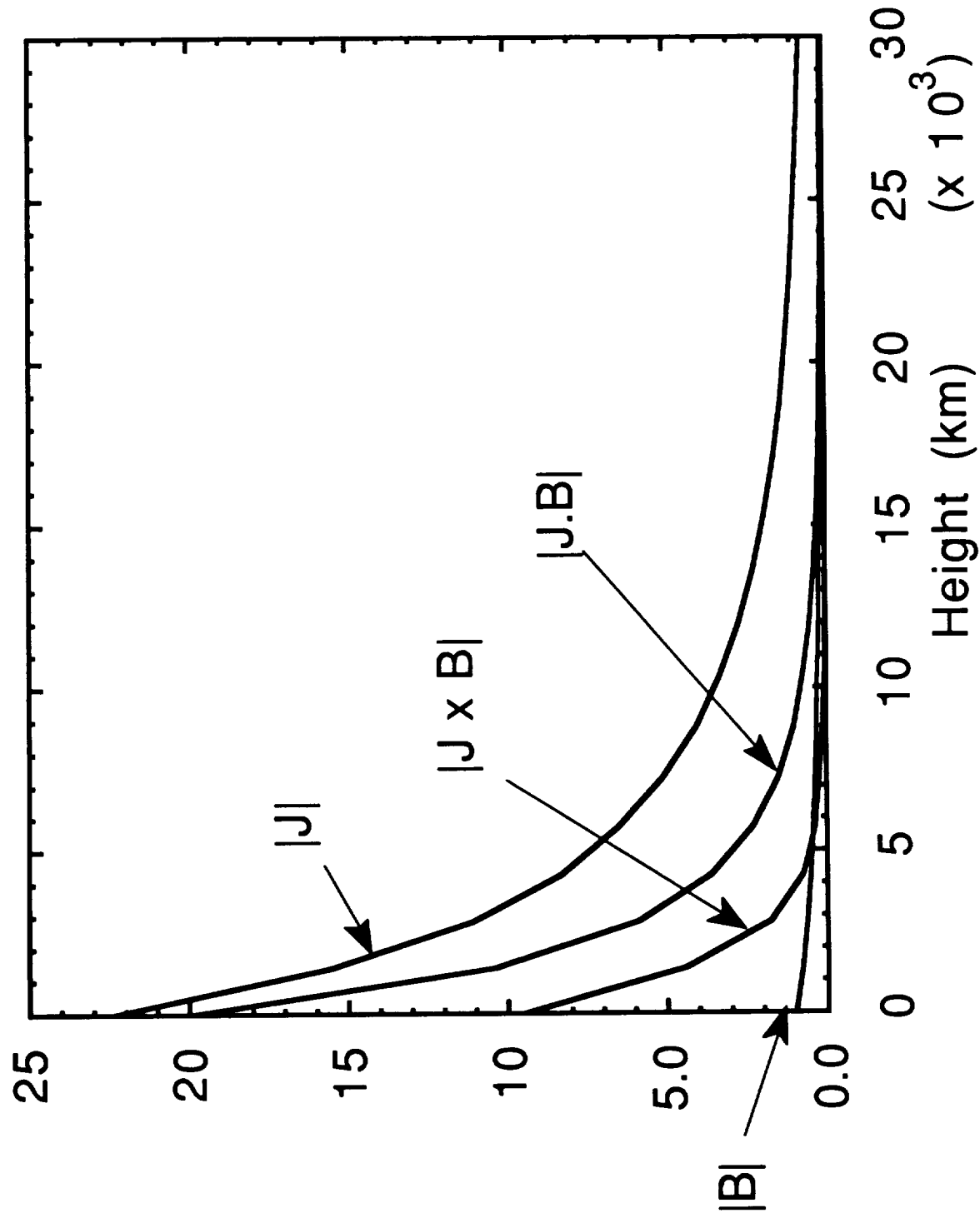




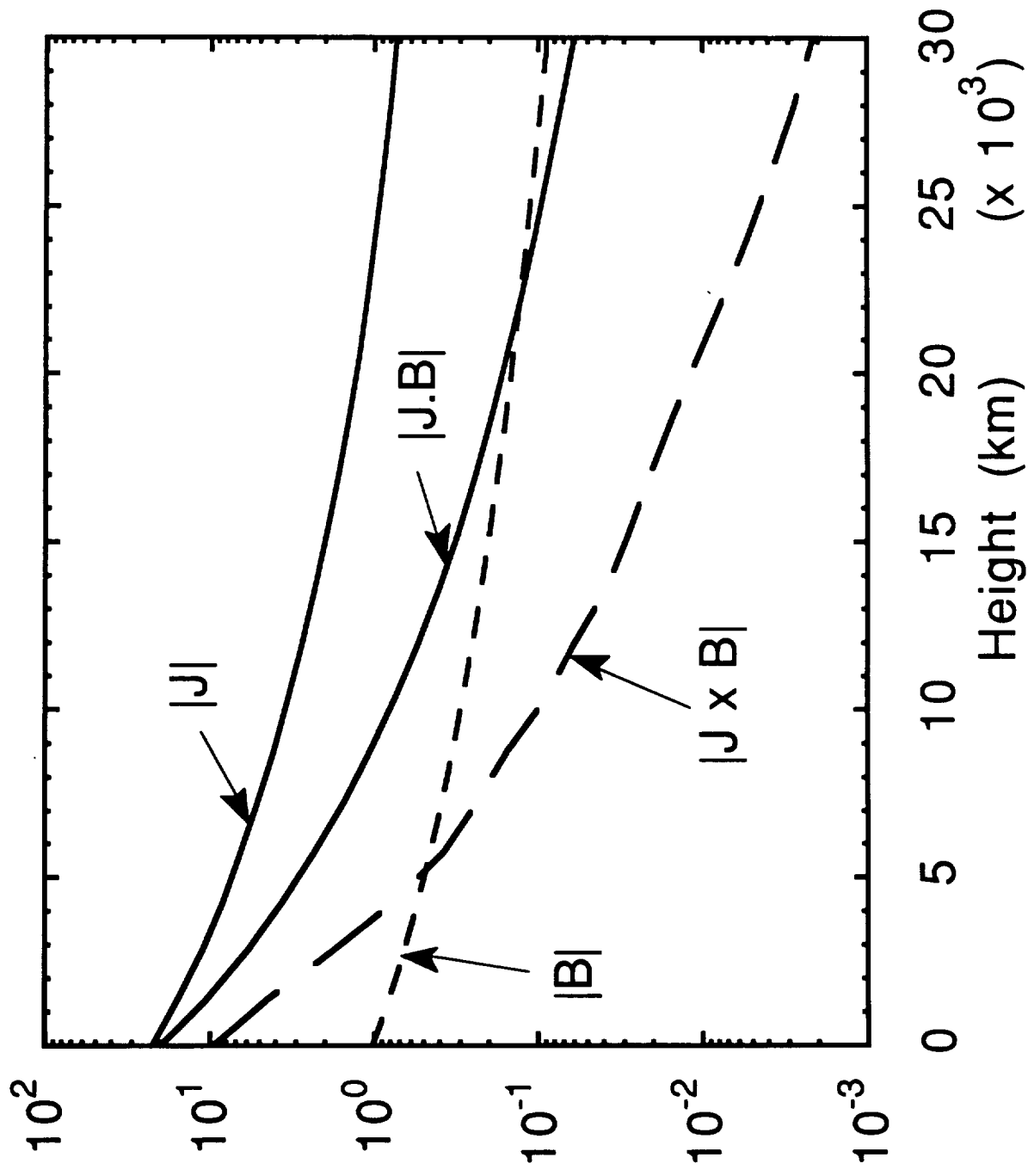
## CHARACTER OF THE SOLUTION

- Coronal field is sheared significantly
- $W_{\text{pot}} \sim 1.44 \times 10^{33}$  ergs
- $W_{\text{ff}} \sim 1.55W_{\text{pot}} = 2.23 \times 10^{33}$  ergs
- Free energy  $\sim 8 \times 10^{32}$  ergs
- The transverse magnetic field from the solution (a predicted quantity) is similar to that from the VM data
- $J_n$  and  $B_n$  match the VM data
- $\mathbf{J} \times \mathbf{B}$  is not small

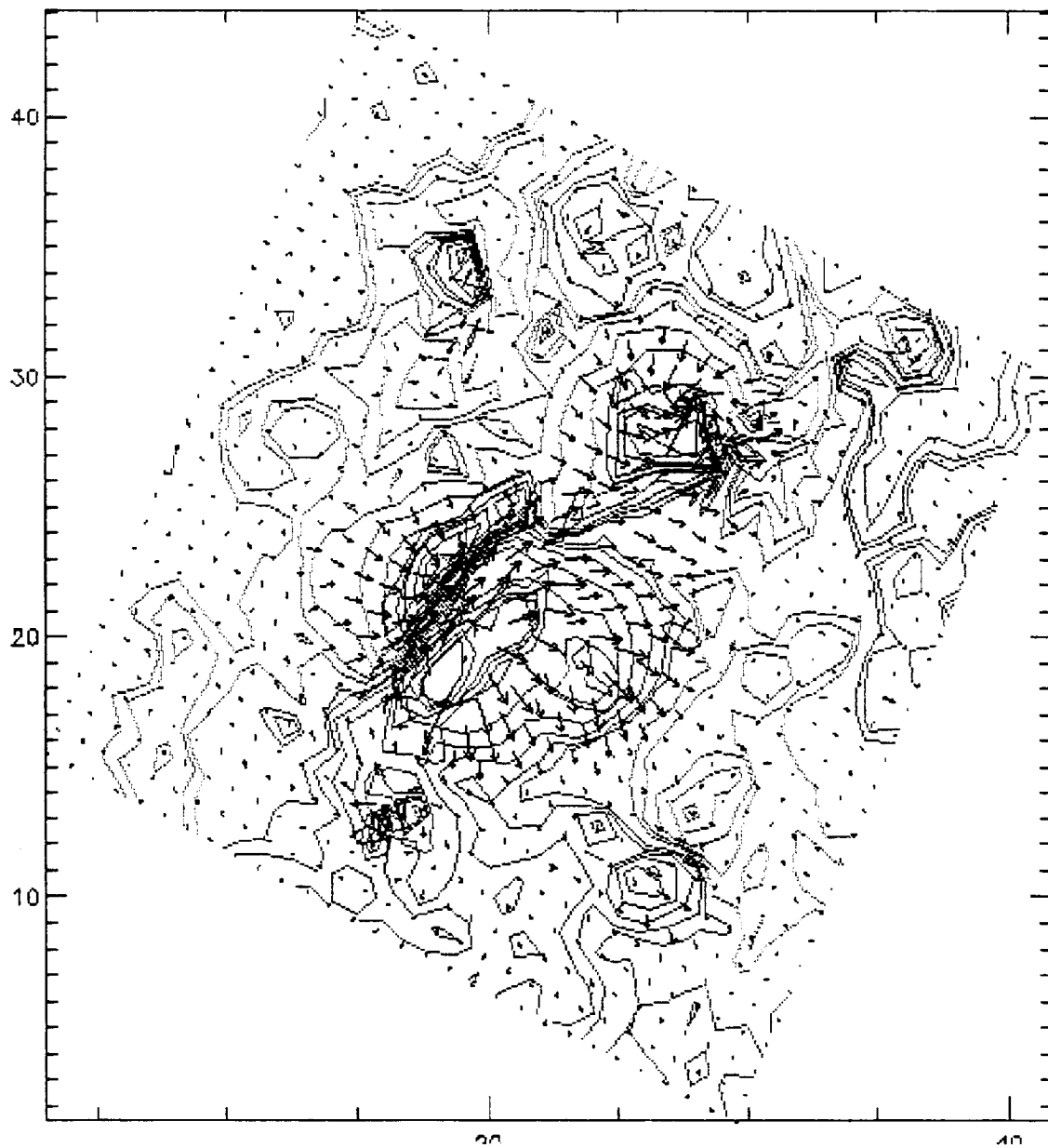
# Variation vs Height



# Variation vs Height



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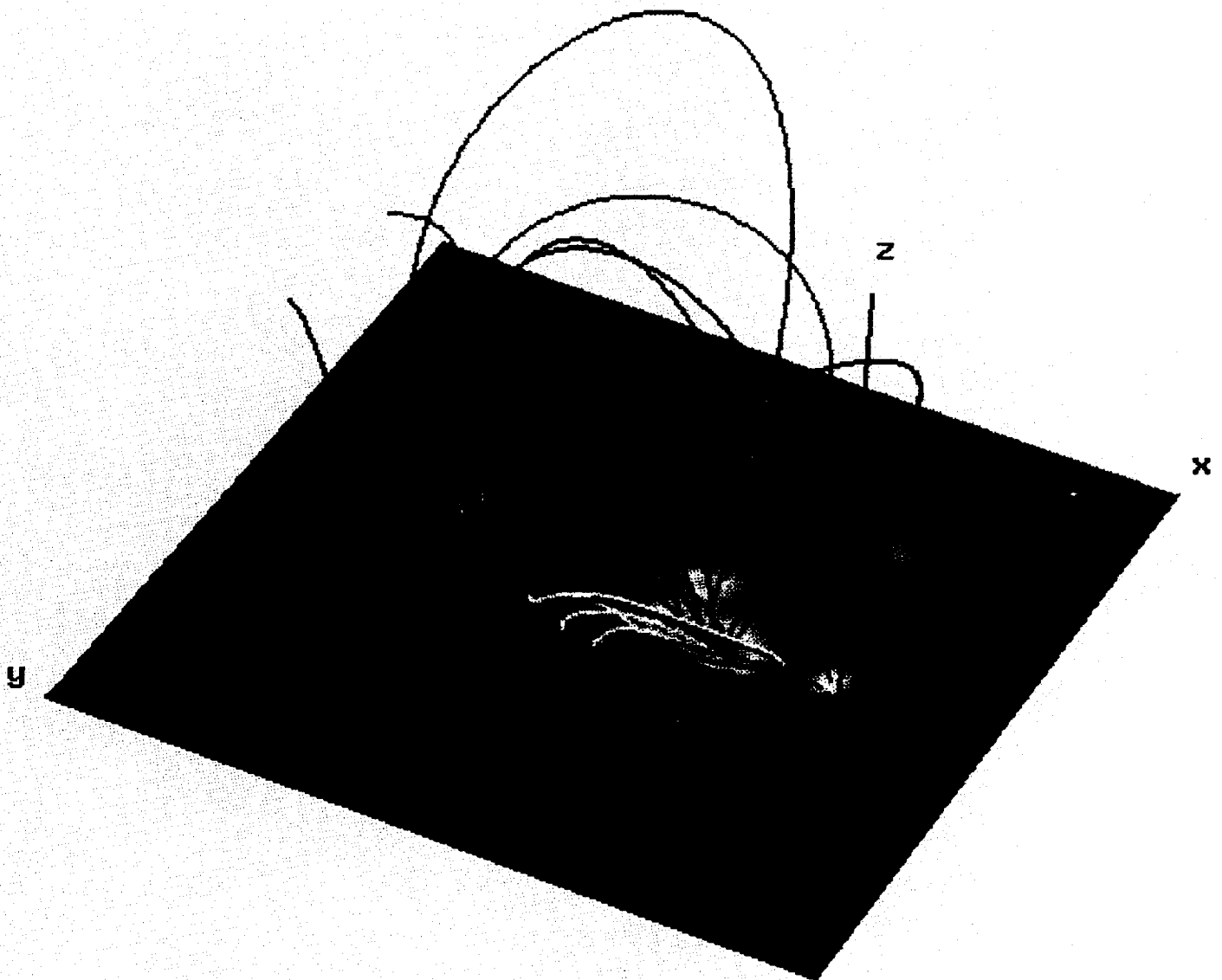


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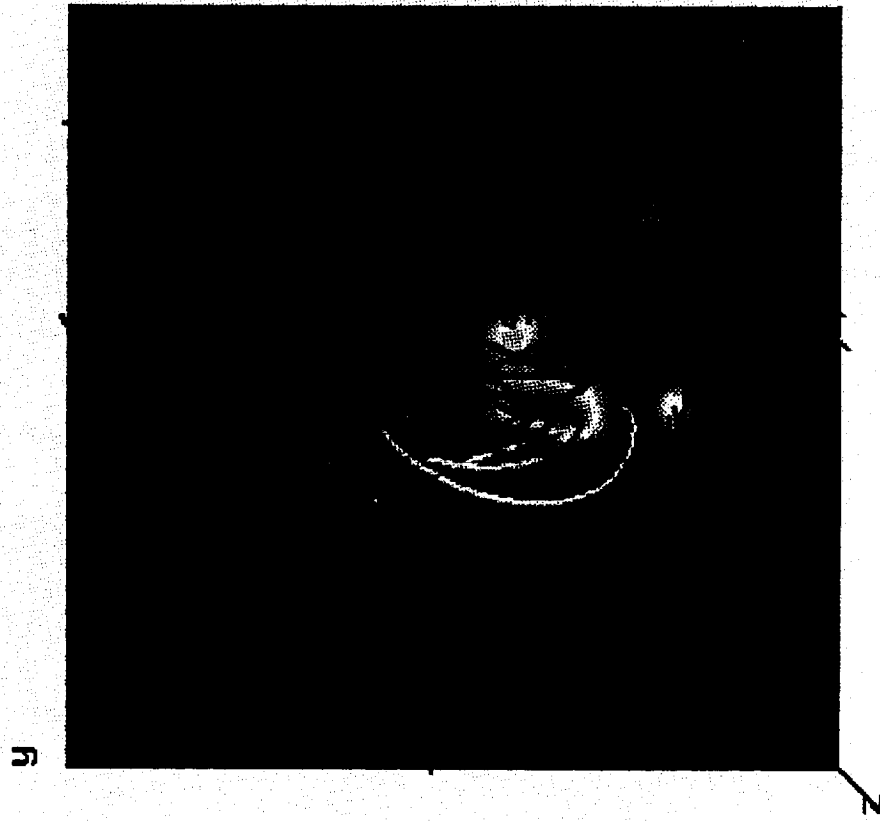
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# "Force-Free" Field

FIGURE 1  
COLOR FIGURE 1



**Field Lines (B)**



**Current Lines (J)**



## CONCLUSION

- The “evolutionary technique” has a sound theoretical basis
- It has given *quantitative* predictions of coronal magnetic fields above an active region
- It can be applied to vector magnetogram measurements (almost) routinely
- The method is quite usable as it stands
- This technique offers exciting new possibilities for detailed analysis of coronal magnetic fields

# IMPROVEMENTS AND FURTHER WORK

- Continue assessment of the effects of measurement errors and lack of resolution on accuracy
- Apply to other vector magnetograms with higher spatial resolution: IVM at IfA; MSFC, APL magnetograms
- Compare the predicted coronal field with other observations (Yohkoh soft X-ray images, CoMStOC, etc.)
- Eliminate periodicity in the transverse plane; use a nonuniform mesh
- Improve the boundary algorithm: reduce  $\mathbf{J} \times \mathbf{B}$  by sacrificing the fit to  $J_n$

